Classical and Quantum Information:

Exploring the abstract limits of Computation and Cognition

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⇨ WHAT IS INFORMATION?

⇨ WHAT DOES INFORMATION LOOK LIKE?

⇨ WHAT CAN YOU DO WITH INFORMATION?

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IN A DYNAMIC WORLD,

_____ acting on _____

So far, we have:

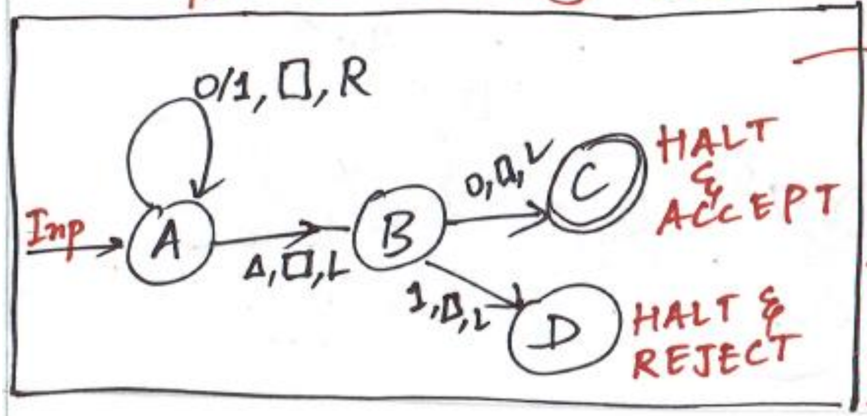
__?__ acting on Information

↳ in general 'A DEVICE'

2

EXAMPLE OF 'A DEVICE'

Turing machine that checks for a binary Even Num.



This TM can be encoded in a binary string.

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Now,

'A DEVICE' acting on INFORMATION

⇩ Becomes

INFORMATION acting on INFORMATION

All we seem to have in our dynamic World!

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? What Makes Information Interesting?

⇒ In the Physical Word:
"Unpredictability"

⇒ In the Abstract World:
"Complexity"

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What Is Complexity?

01010101	v/s	0110 0110
(A)		(B)

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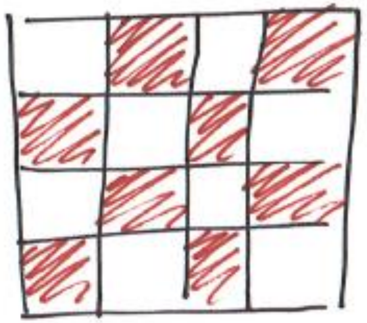
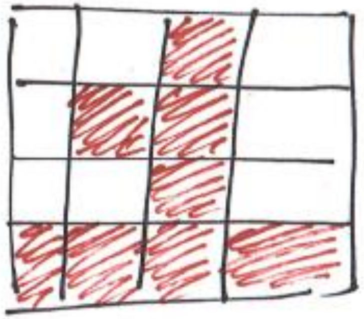
Which One Is More Complex?



(A)



(B)

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Which Is More Complex?

Addition v/s Multiplication

Is-A-Factor(x) v/s Is-A-Prime (x)

Determinant (A) v/s Inverse (A)

<But, why ??>

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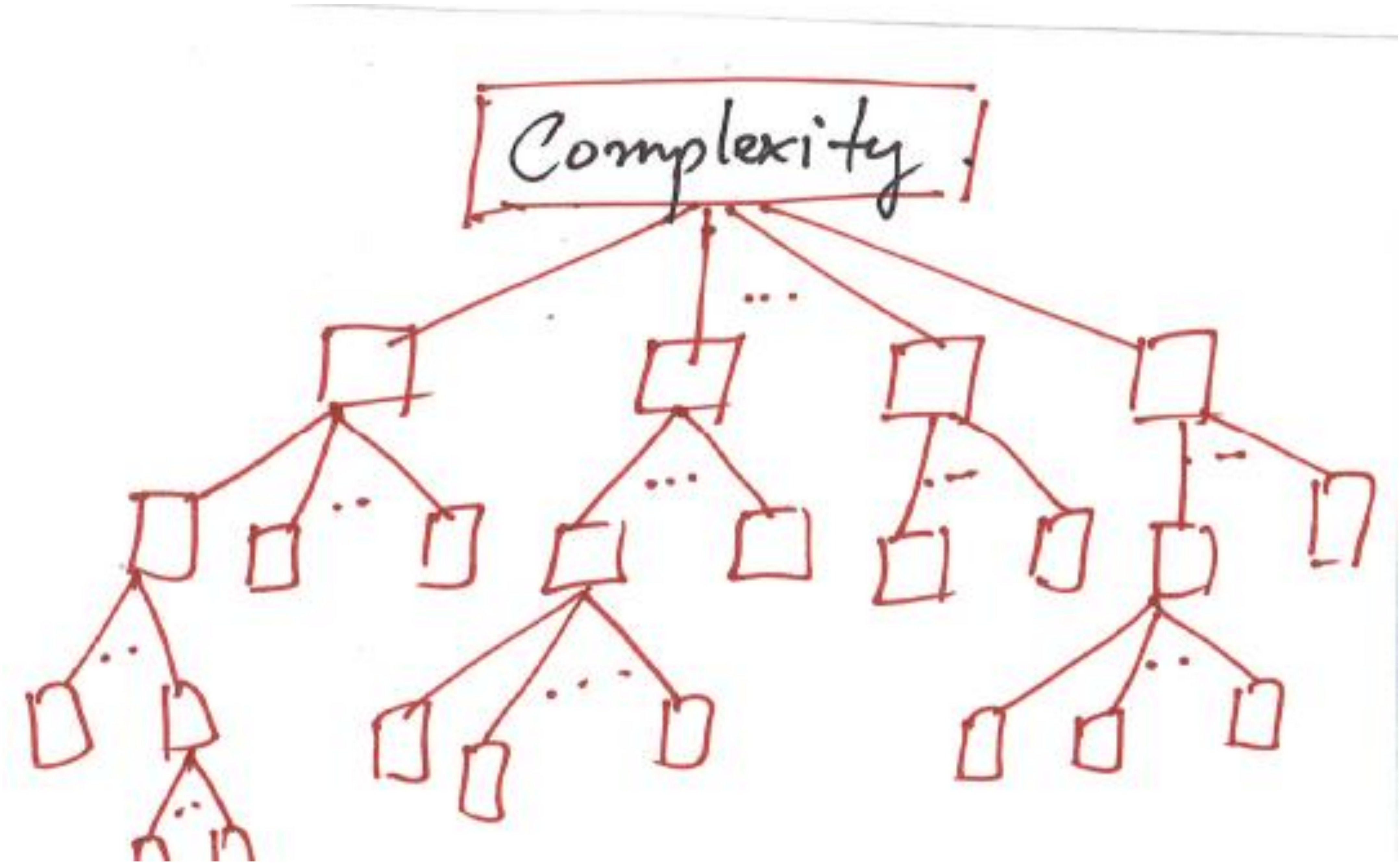

Complexity

Part I: Classical Information

WHAT IS THE STRENGTH OF TURING MACHINES?

Can compute any Recursive function

i.e.

Can decide any Recursive Language

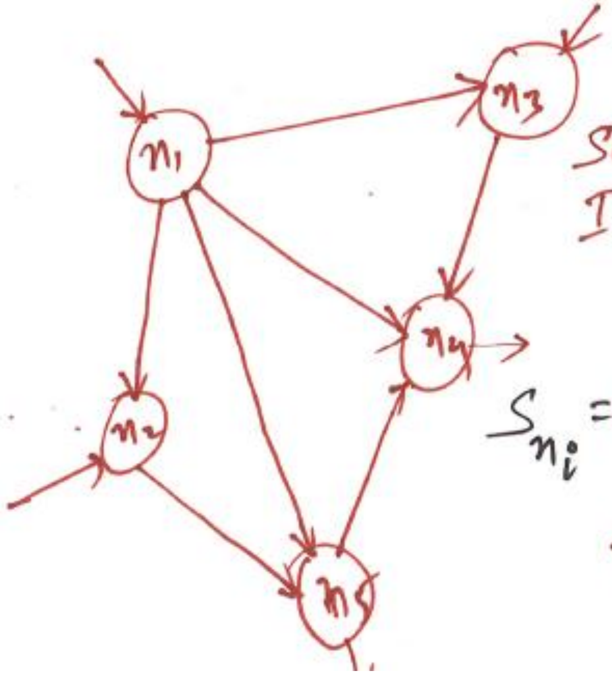
Computational Power of RNNs.

RNN[Q]: Can decide all Turing decidable Languages

RNN[R]: Can decide any Language, but in exponential time.

But RNNs are just one of Many Many Neural Architectures!!

From Computational Capabilities POV, the number of fundam-entally different models is very few!



Simple Internal States.

$$S_{n_i} = \sigma\left(\sum_{j=1}^{n} w_j x_j - w_0\right)$$

σ → transfer function.

WHAT DECIDES THE COMPLEXITY?

- How states of neurons are computed?
- Cyclic / Acyclic Networks
- Symmetric / Asymmetric Connections
- Continuous / Discrete Update Methods
- ...... So on ...

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LOADING PROBLEM

GIVEN:

1) Input
$\{(x_1, b_1), \dots (x_m, b_m)\}$
$x_i \in \{0,1\}^n, b_i \in \{0,1\}$

2) Directed Acyclic Graph

To COMPUTE:

Assignment of weights, such that,

$f(x_i) = b_i$
$\forall i = 1 \dots m$

NP-COMPLETE!

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NEURAL CIRCUIT MINIMISATION

Is there a circuit with at most K neurons, such that 'f' computed by it has
$f(x_i) = b_i, \forall i = 1 \dots m$.

NP-Complete!

→ A slightly relaxed version? ⇒ with bounded poly. depth?

Still Intractable!

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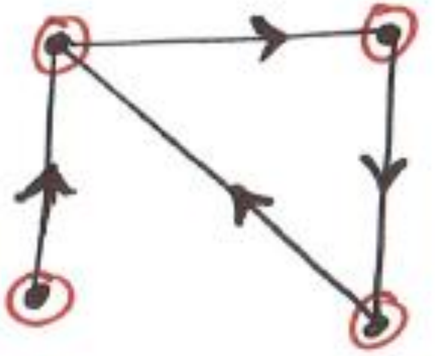
Part I: Quantum Information

WHAT IS SPECIAL ABOUT THIS GRAPH?



(Never happens in Nature ....)

Fundamental Law of Physics :-

Unique Future & Unique Past

BUT WHAT ABOUT QUANTUM PHYSICS?

A QUBIT.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

(α, β →) Complex amplitudes such that:

$$|\alpha^2 + \beta^2| = 1.$$

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CLASSICAL COMPUTATION:

⇒ Set Theory – dealing with mainly Sets & Subsets.

QUANTUM COMPUTATION:

⇒ Spaces & Projections – dealing with subspaces, eigen sub-spaces, etc.

→ MATRICES ⇒ LINEAR ALGEBRA!!

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Evolution Of a Quantum System

S_0 → start state – Superposition of pure states

M → Matrix with system evolution rules → Unitary

$$S_1 = MS_0$$

$$S_t = MS_{t-1} = M^t S_0.$$

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OBSERVABLES & HERMITIAN MATRIX. OPERATOR

A → observable. (position / momentum / Spin) Hermitian Operator.

$|a\rangle$ → eigenket / eigen Vector. → state of Q. system

$|a\rangle$ → eigen value

$$\boxed{A|a\rangle = a|a\rangle}$$

$$AB \neq BA.$$

→ Measurements are Non-commutative.

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MOST POPULAR QUANTUM ALGORITHMS

1. SHOR'S ALGORITHM
   (TO FACTOR NUMBERS)
   → poly.(log(N)) → where 'N' is input no.
2. GROVER'S ALGORITHM
   (SEARCHING UNSTRUCTURED DATABASES)

ADIABATIC OPTIMIZATION FOR NP-HARD PROBLEMS

⇒ Gradually changing conditions allow system to adapt its configuration

⇒ To solve for optimisation problems.

(closely related to Quantum Annealing)

QUANTUM FAULT TOLERANCE.

A quantum computer with noise can accurately simulate an ideal quantum Computer, (provided noise is below a threshold).

In practise, errors can be controlled as number of qubits scales up.

QUANTUM FOURIER TRANSFORM

→ Linear Transformation on 'Qubits'.

→ Quantum Analogue of DFT.

→ Used Extensively; SHOR'S Algo, etc.

Input: 'n'-bits:

C. DFT → $O(n.2^n)$ gates ; QFT → $O(n^2)$ gates.

QUANTUM ALGORITHMS FOR MACHINE LEARNING.

Quantum PCA.

Quantum K-neighbours (nearest)

Quantum RL.

Quantum Netflix Recommen-dation 1

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ADIABATIC ALGORITHM.

↳ QUANTUM ANALOGUE OF SIMULATED ANNEALING.

(For global minima of a discrete search space).

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QUANTUM SAMPLING

Important for probabilistic Learning, Deep Learning, etc..

To find optimal control parameters that best represent the empirical distribution of a given data-set.

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QUANTUM TUNNELING & ML.

"from dual nature of particles".

↳ None of the possibilities have zero probability!

OPTIMISATION ⇒ FINDING GLOBAL OPTIMA!

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WHERE IN NATURE ?

- → PHOTOSYNTHESIS.
- → OLFACTION.
- → NAVIGATION IN EUROPEAN ROBINS.
- → BACTERIA – Q. RANDOM WALK
- → SO... ON....

OTHER THAN LEARNING ALGORITMS.

- → PRIVACY in Database Query, Search engines.
- → Quantum Finance – to replace computational Finance for faster Analytic solvers.
- → Piracy → of software, etc.

QUANTUM IS MYSTERIOUS....

MIND IS MYSTERIOUS.....

THEREFORE, THEY MUST BE RELATED SOMEHOW ???  :P

WHAT IF ? (o_O)

- → Free-will discussions!
- → thousands of Brain Scans!

THIS WAS

QUANTAMIZED CLASSICAL COMPUTATION...
& CLASSICAL AI.....

WHAT ABOUT THE ACTUAL QUANTUM COMPUTATION & QUANTUM AI ??...

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QUANTUM LOGIC GATES.

SQUARE ROOT OF 'NOT' GATE.

$$\sqrt{NOT} = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$$

$$\sqrt{NOT}\sqrt{NOT} = NOT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

MORE LOGIC ~ MORE VARIETY OF EXP.

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PATTERN RECOGNITION

classical Info. & classical computers	Classical Info. & Quantum Computers
Quantum Info. & classical computers	Quantum Info. & Quantum Computerns

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Remembering Stephen Hawking….

Crazy possibility of Ultra-Computation with Black holes ☺

BLACK HOLE INFORMATION PARADOX.

Hawking Radiation

≡ black body radiation near event horizon due to Quantum Effects.

$$R = \frac{2MG}{c^2}$$

BLACK HOLES &
COMPUTATION.

Connecting limits of Geometry
Of Space-Time to
computational Capacity.

$$t \propto \frac{1}{E}$$

NEED FOR QUANTUM GRAVITY...

Questions?

Apologies for the terrible handwriting in the slides! ☺

Contact: gm.sushravya@gmail.com

I thoroughly enjoyed answering fantastic questions from my brilliant audience..
Thank you! I hope you found some interesting ideas to play around with ☺..
Feel free to contact for any doubts or further readings!
Best,
Shravya